Analog Electronic

ENEE236

FET Amplifiers
ac small signal analysis

Instructor: Nasser Ismail

Definition: Transconductance g_m

For JFETs and DMOSFETs

$$g_m = \frac{\partial \mathrm{I_D}}{\partial \mathrm{V_{GS}}} = \frac{2I_{DSS}}{|V_P|}\left[1 - \frac{V_{GS}}{V_P}\right]$$

$$g_m = g_{m0}\left[1 - \frac{V_{GS}}{V_P}\right] = g_{m0}\sqrt{\frac{I_D}{I_{DSS}}}$$

$$g_{m0} = \frac{2I_{DSS}}{|V_P|}$$

For EMOSFET

$$I_D = \mathrm{K}\left(V_{GS} - V_{GS(TH)}\right)^2$$

$$K = \frac{I_D}{\left(V_{GS} - V_{GS(TH)}\right)}$$

$$g_m = \frac{\partial \mathrm{I_D}}{\partial \mathrm{V_{GS}}} = 2\mathrm{K}\left(V_{GS} - V_{GS(TH)}\right)$$

$$\left(V_{GS} - V_{GS(TH)}\right) = \sqrt{\frac{\mathrm{I_D}}{\mathrm{K}}}$$

$$\therefore g_m = 2K\sqrt{\frac{\mathrm{I_D}}{\mathrm{K}}} = 2\sqrt{\frac{\mathrm{I_D}K^2}{\mathrm{K}}} = 2\sqrt{\mathrm{I_D}K}$$

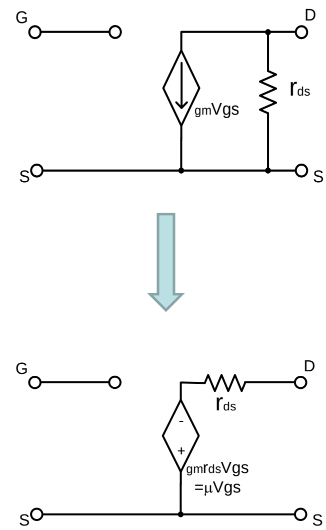
AC Small Signal Equivalent Circuit (MODEL Valid for all FET Types)

- In ac

$$g_m = \frac{i_d}{v_{gs}} \Rightarrow i_d = g_m v_{gs}$$

- Or

$$\mu = g_m r_{ds} \quad \text{- amplification factor}$$



FET Impedence

Input impedance:

$$Z_i = \infty\,\Omega$$

Output Impedance:

$$Z_o = r_{ds} = \frac{1}{y_{os}} \quad \text{where} \quad r_{ds} = \left.\frac{\Delta V_{DS}}{\Delta I_D}\right|_{V_{GS} = \text{constant}}$$

y_{os}= admittance parameter listed on FET spec sheets

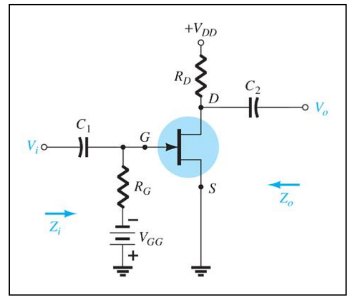
Common-Source (CS) Fixed-Bias

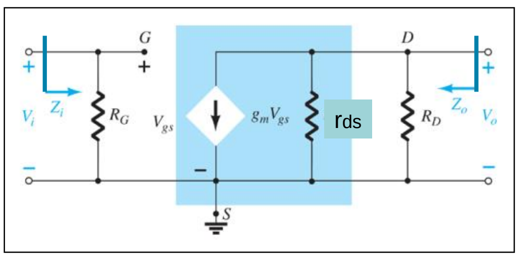
The input is applied to the gate and the output is taken from the drain

There is a 180° phase shift between the circuit input and output

<u>To construct ac ss equivalent circuit</u>

1) C1 & C2 are replaced by short
2) V_{DD}=0 V (short)
3) FET ac ss MODEL





Calculations

Input impedance:

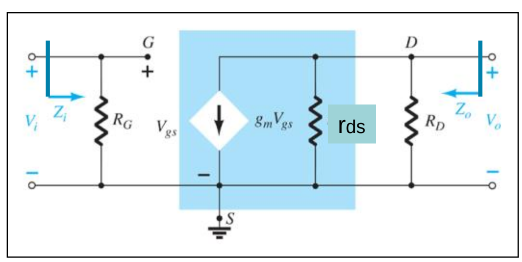
$$Z_i = R_G$$

Output impedance:

$$Z_o\Big|_{Vi=0} = R_D || r_{ds}$$

$$Z_o\Big|_{Vi=0} \cong R_D\Big|_{r_{ds} \geq 10 R_D}$$

Voltage gain:



$$V_i = V_{gs}$$

$$V_o = V_{ds}$$

$$A_v = \frac{V_o}{V_i} = \frac{V_{ds}}{V_{gs}}$$

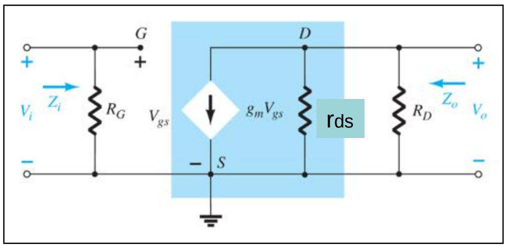
$$V_{ds} = -g_m V_{gs} (r_{ds} || R_D)$$

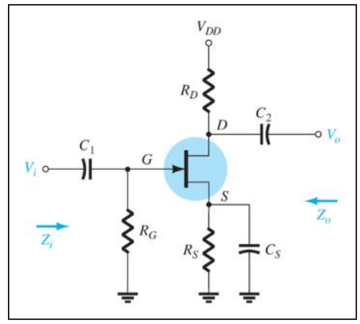
$$A_v = \frac{V_{ds}}{V_{gs}} = -g_m (r_{ds} || R_D)$$

$$A_v = \frac{V_o}{V_i} = -g_m R_D\Big|_{r_{ds} \geq 10 R_D}$$

Common-Source (CS) Self-Bias

This is a common-source amplifier configuration, so the input is applied to the gate and the output is taken from the drain.





There is a 180° phase shift between input and output.

Calculations

Input impedance:

$$Z_i = R_G$$

Output impedance:

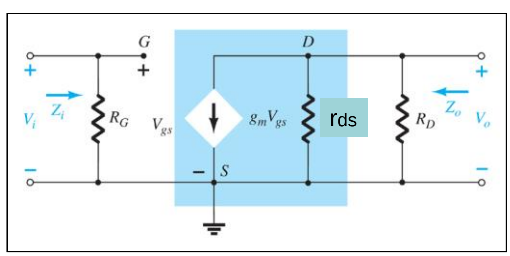
$$Z_o = r_{ds} || R_D$$

$$Z_o \cong R_D \Big|_{r_{ds} \geq 10 R_D}$$

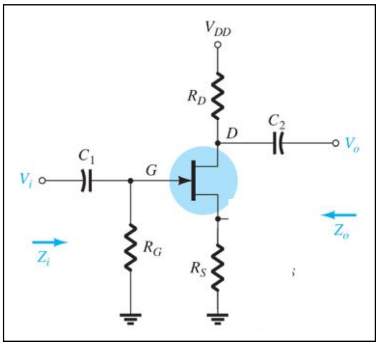
Voltage gain:

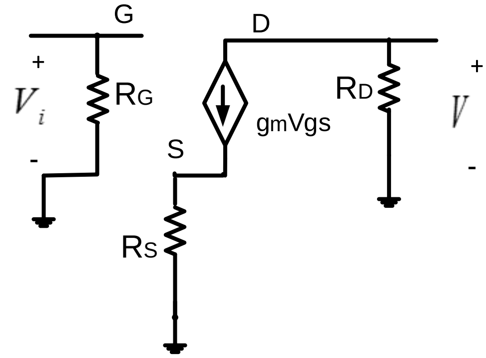
$$A_v = -g_m (r_{ds} || R_D)$$

$$A_v = -g_m R_D \Big|_{r_{ds} \geq 10 R_D}$$



Common-Source (CS) Self-Bias
Effect of Rs (ignore rds)





$$A_v = \frac{V_o}{V_i}$$

$$V_o = -g_m V_{gs}(R_D)$$

$$V_s = g_m V_{gs}(R_S)$$

$$V_g = V_i$$

$$V_{gs} = V_g - V_S = V_i - g_m V_{gs} R_S$$

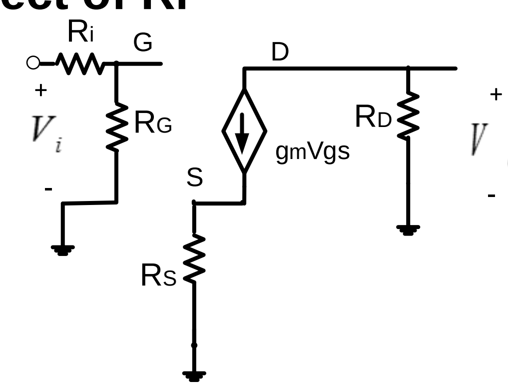
$$\Rightarrow V_i = V_{gs} + g_m V_{gs} R_S$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_m V_{gs} R_D}{V_{gs} + g_m V_{gs} R_S}$$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

Gain is reduced due to R_S

Common-Source (CS) Self-Bias
Effect of Ri



$$A_v = \frac{V_o}{V_i}$$

$$V_o = -g_m V_{gs}(R_D)$$

$$V_s = g_m V_{gs}(R_S)$$

$$V_g = \frac{R_G}{R_G + R_i} V_i$$

$$V_{gs} = V_g - V_S = \frac{R_G}{R_G + R_i} V_i - g_m V_{gs} R_S$$

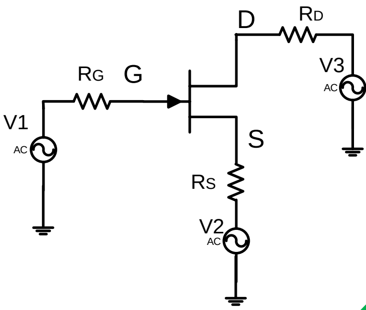
$$\Rightarrow V_i = V_{gs}(1 + g_m R_S) \frac{R_G + R_i}{R_G}$$

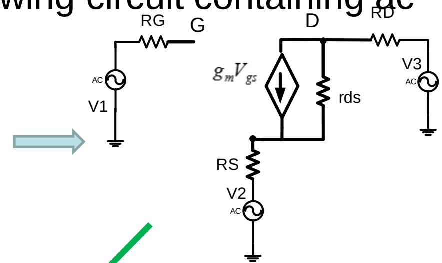
$$A_v = \frac{V_o}{V_i} = \frac{-g_m R_D}{1 + g_m R_S} \frac{R_G}{R_G + R_i}$$

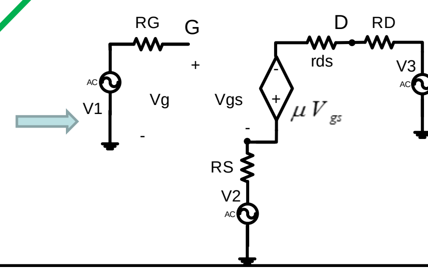
Gain is reduced more due to R_i

Impedance Reflection

- Consider the following circuit containing ac sources V1,V2,V3



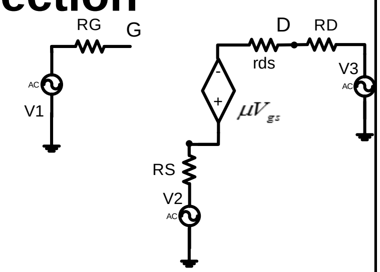




with source transformation

$\mu = g_m r_{ds}$ - amplification factor

Impedance Reflection



KVL for the drain - source loop

$$V_3 - i_D R_D - i_D r_{ds} - i_D R_S + \mu V_{gs} - V_2 = 0 \text{..................}(1)$$

but

$$V_{gs} = V_g - V_S = V_g - (i_D R_S + V_2) \text{...........................}(2)$$

substituting (2) in (1) yields:

$$V_3 - i_D R_D - i_D r_{ds} - i_D R_S + \mu(V_g - (i_D R_S + V_2)) - V_2 = 0$$

$$V_3 - i_D R_D - i_D r_{ds} - i_D R_S + \mu V_g - \mu i_D R_S - \mu V_2 - V_2 = 0$$

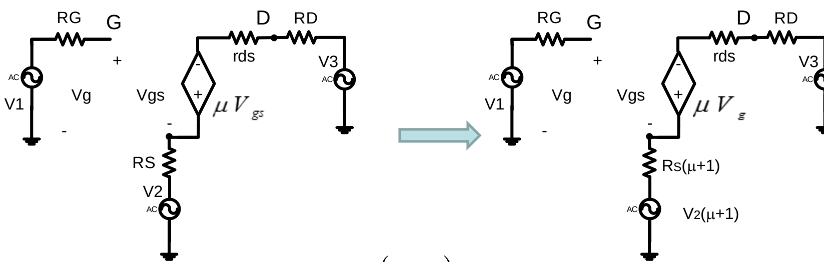
$$V_3 - i_D R_D - i_D r_{ds} - i_D R_S(\mu + 1) + \mu V_g - V_2(\mu + 1) = 0$$

$$i_D R_D + i_D r_{ds} + i_D R_S(\mu + 1) = V_3 + \mu V_g - V_2(\mu + 1)$$

$$V_3 - i_D R_D - i_D r_{ds} - i_D R_S + \mu(V_g - (i_D R_S + V_2)) - V_2 = 0$$

$$i_D = \frac{V_3 + \mu V_g - V_2(\mu + 1)}{R_D + r_{ds} + R_S(\mu + 1)} \text{.....................}(3)$$

(3) is the drain Equivalent circuit equation



$$i_D = \frac{V_3 + \mu V_g - V_2(\mu + 1)}{R_D + r_{ds} + R_S(\mu + 1)} \text{....................(3)}$$

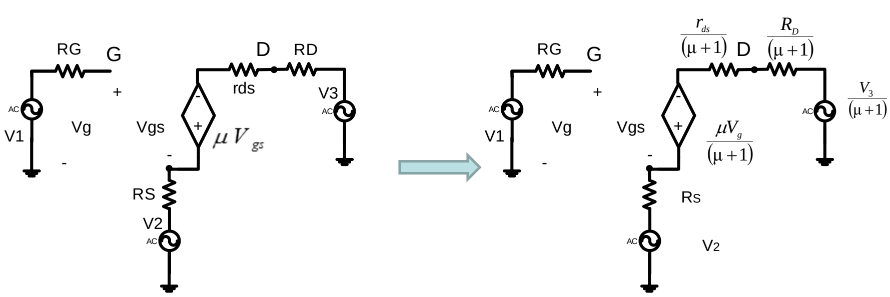
(3) is the drain Equivalent circuit equation

Reflection from Source to Drain

$\mu V_{gs} \Rightarrow \mu V_g$

$R_S \Rightarrow R_S(\mu + 1)$

$V_2 \Rightarrow V_2(\mu + 1)$



divide eq. (3) by $(\mu + 1)$

$$i_D = \frac{\frac{V_3}{(\mu+1)} + \frac{\mu V_g}{(\mu+1)} - V_2}{\frac{R_D}{(\mu+1)} + \frac{r_{ds}}{(\mu+1)} + R_S} \text{....................(4)}$$

(4) is the source equivalent circuit equation

Reflection Drain to Source

$\mu V_{gs} \Rightarrow \frac{\mu V_g}{(\mu + 1)}$

$R_D \Rightarrow \frac{R_D}{(\mu + 1)}$

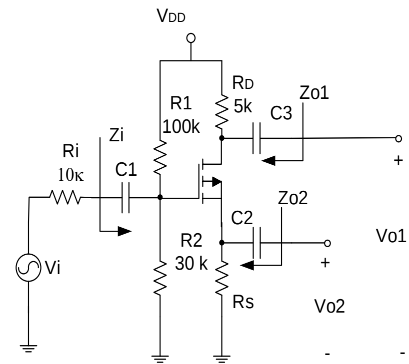
$r_{ds} \Rightarrow \frac{r_{ds}}{(\mu + 1)}$

Example: Phase Splitting circuit

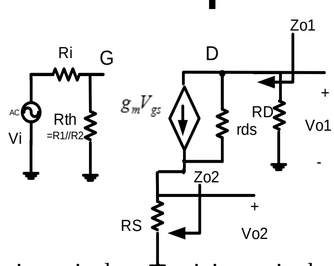
- Two outputs:
  - Vo1 from drain
  - Vo2 from source

Find A_V, A_I, Z_{O1}, Z_{O2} and Z_I

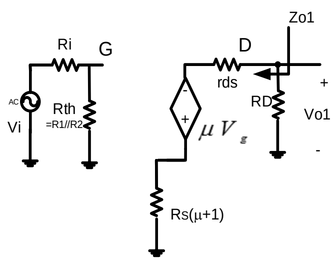
$r_{ds} = 100\ k\Omega$
$g_m = 1\ mS$



Solution: ac ss equivalent circuit



1) To Find Z_{O1}, V_{O1} Drain equivalent circuit is required since both of these quantities are seen from the drain

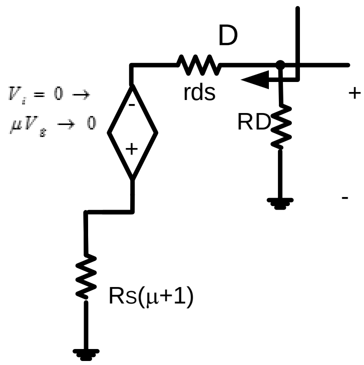


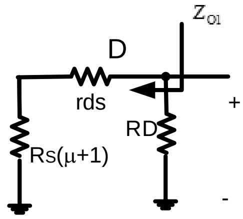
$$V_{o1} = \frac{R_D}{R_D + r_{ds} + R_S(\mu+1)}\left(-\mu V_g\right)$$

$$V_g = V_i \frac{R_{th}}{R_{th} + R_i}$$

$$Av = \frac{V_{o1}}{V_i} = (-\mu)\frac{R_D}{R_D + r_{ds} + R_S(\mu+1)} \cdot \frac{R_{th}}{R_{th} + R_i}$$

2) To Find $Z_{O1}\Big|_{\substack{Vi=0\\Vg=0}}$

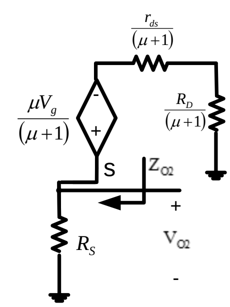




$$Z_{O1}\Big|_{\substack{Vi=0\\Vg=0}} = R_D // \left[r_{ds} + R_s(\mu+1) \right]$$

Solution: continued

3) To Find Z_{O2}, V_{O2} Source equivalent circuit is required since both of these quantities are seen from the source

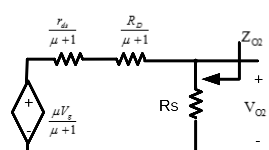


Reflection Drain to Source

$$\mu V_{gs} \Rightarrow \frac{\mu V_g}{(\mu+1)}$$

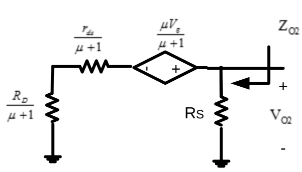
$$R_D \Rightarrow \frac{R_D}{(\mu+1)}$$

$$r_{ds} \Rightarrow \frac{r_{ds}}{(\mu+1)}$$



$$V_{o2} = \frac{R_{|S}}{\frac{R_D + r_{ds}}{(\mu+1)} + R_S} \left(\frac{\mu V_g}{(\mu+1)} \right)$$

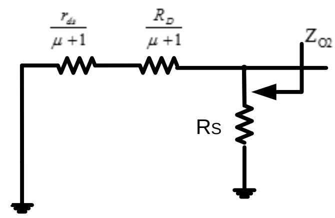
$$V_g = V_i \frac{R_{th}}{R_{th} + R_i}$$



$$A_{v2} = \frac{V_{o2}}{V_i} = \frac{\mu}{(\mu+1)} \frac{R_{|S}}{\frac{R_D + r_{ds}}{(\mu+1)} + R_S} \cdot \frac{R_{th}}{R_{th} + R_i}$$

Solution: continued

4) To Find $Z_{O2}\big|_{\substack{Vi=0\\Vg=0}}$



$$Z_{O2}\big|_{\substack{Vi=0\\Vg=0}} = R_S \,//\left[\frac{r_{ds}+R_D}{(\mu+1)}\right]$$

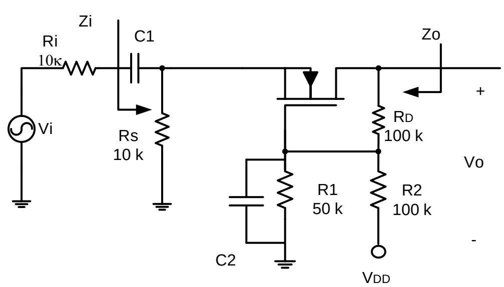
$$Z_{O2}\big|_{\substack{Vi=0\\Vg=0}}\Big|_{r_{ds}\to\infty} = R_S \,// \frac{1}{g_m}$$

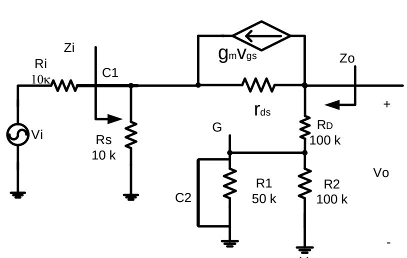
since $\lim_{rds\to\infty} \frac{rds+R_D}{\mu+1} = \lim \frac{\frac{rds}{rds}+\frac{R_D}{rds}}{\frac{g_m rds}{rds}+\frac{1}{rds}}$

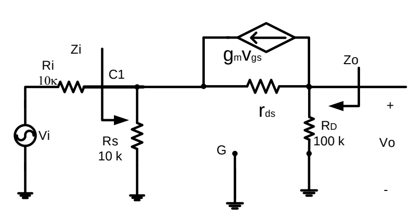
$= \lim_{rds\to\infty} \frac{1+\frac{R_D}{rds}}{g_m+\frac{1}{rds}} = \frac{1}{g_m}$

$$Z_i = R_{th} = R_1 \,//\, R_2$$

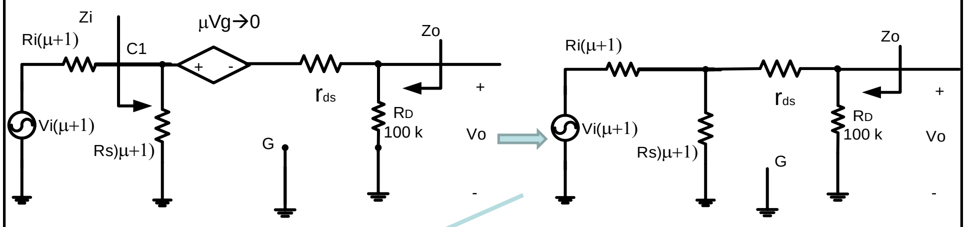
Common Gate Amplifier

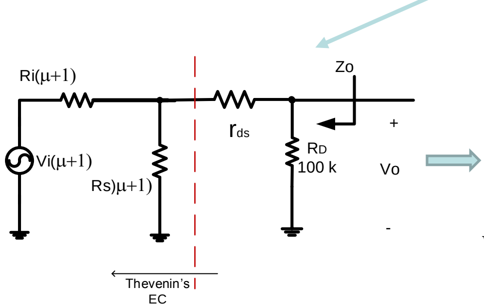


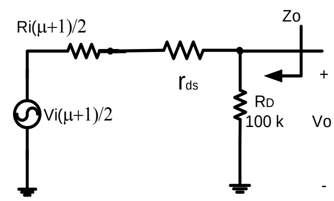




Drain Equivalent circuit to find Vo and Zo



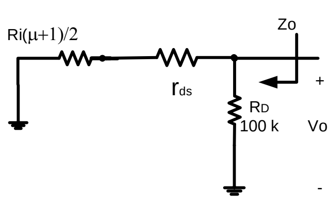




$$V_o = \frac{R_D}{R_D + r_{ds} + \left(R_S // R_i\right)(\mu+1)}\left(\frac{(\mu+1)}{2}V_i\right)$$

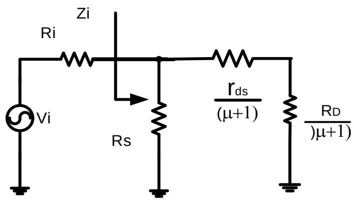
$$Av = \frac{V_o}{V_i} = \frac{R_D}{R_D + r_{ds} + 5k(\mu+1)}\left(\frac{(\mu+1)}{2}\right)$$

- Drain Equivalent circuit to find Vo and Zo



$$Z_O\Big|_{Vi=0} = R_D // \left(r_{ds} + \frac{R_i(\mu+1)}{2}\right)$$

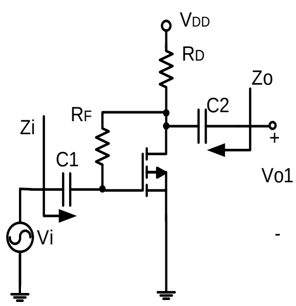
- To find Zi source equivalent circuit is needed

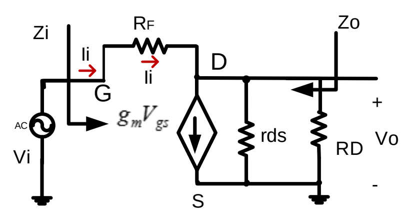


$$Z_i = R_S // \left[\frac{r_{ds} + R_D}{(\mu + 1)}\right]$$

$$Z_i\Big|_{r_{ds}\to\infty} = R_S // \frac{1}{g_m}$$

Drain Feedback Configuration (self study)





$$I_i = g_m V_{gs} + \frac{V_o}{R_D // r_{ds}}$$

$$V_{gs} = V_i$$

$$I_i = g_m V_i + \frac{V_o}{R_D // r_{ds}}$$

$$I_i - g_m V_i = \frac{V_o}{R_D // r_{ds}}$$

$$V_o = (I_i - g_m V_i)(R_D // r_{ds})$$

also

$$I_i = \frac{V_i - V_O}{R_F}$$

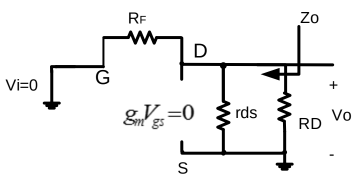
$$= \frac{V_i - ((I_i - g_m V_i)(R_D // r_{ds}))}{R_F}$$

$$I_i R_F \quad = V_i - ((I_i - g_m V_i)(R_D // r_{ds}))$$

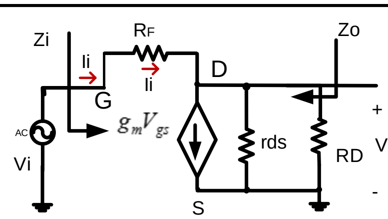
$$V_i[1 + g_m(R_D // r_{ds})] = I_i[R_F + (R_D // r_{ds})]$$

$$\therefore$$

$$Z_i = \frac{V_i}{I_i} = \frac{[R_F + (R_D // r_{ds})]}{[1 + g_m(R_D // r_{ds})]}$$



$$Z_o\big|_{vi=0} = R_D // r_{ds} // R_F$$



$$I_i = g_m V_{gs} + \frac{V_o}{(R_D // r_{ds})}$$

$$V_{gs} = V_i \quad \textit{also} \quad I_i = \frac{V_i - V_O}{R_F}$$

$$\frac{V_i - V_O}{R_F} = g_m V_{gs} + \frac{V_o}{(R_D // r_{ds})}$$

$$\frac{V_i}{R_F} - \frac{V_O}{R_F} = g_m V_i + \frac{V_o}{(R_D // r_{ds})}$$

$$\frac{V_i}{R_F} - g_m V_i = \frac{V_o}{(R_D // r_{ds})} + \frac{V_O}{R_F}$$

$$V_i\left(\frac{1}{R_F} - g_m\right) = V_o\left(\frac{1}{(R_D // r_{ds})} + \frac{1}{R_F}\right)$$

$$A_V = \frac{V_o}{V_i} = \frac{\left(\frac{1}{R_F} - g_m\right)}{\left(\frac{1}{(R_D // r_{ds})} + \frac{1}{R_F}\right)}$$

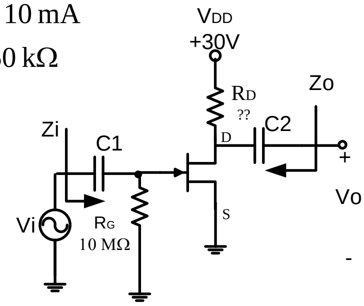
FET Amplifier Design (Important)

- Design a fixed bias network such that the ac voltage gain $|Av| = 10$, i.e. find value of R_D

$V_P = -4\text{ V}$

$I_{DSS} = 10\text{ mA}$

$r_{ds} = 50\text{ k}\Omega$



Solution

$V_{GS} = V_G - V_S = 0V$

$$I_D = I_{DSS}\left(1 - \frac{0}{-4}\right)^2 = I_{DSS} = 10mA$$

For JFETs

$$g_m = \frac{2I_{DSS}}{|V_P|}\left[1 - \frac{V_{GS}}{V_P}\right]$$

$$= \frac{2(10\text{mA})}{|-4|}\left[1 - \frac{0}{-4}\right] = 5\text{ m}S$$

$V_{gs} = V_i$

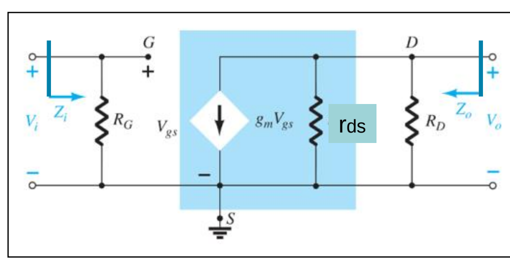
$$A_v = \frac{V_o}{V_i}$$

$V_o = -g_m V_{gs}(r_{ds}||R_D)$

$V_o = -g_m V_i(r_{ds}||R_D)$

$$|A_v| = \left|\frac{V_o}{V_i}\right| = |-g_m(r_{ds}||R_D)|$$

ac ss equivalent circuit



Since Av & gm are known , then

$$|A_v| = \left|\frac{V_o}{V_i}\right| = |-g_m(r_{ds}||R_D)| = 10$$

$$\therefore (r_{ds}||R_D) = \frac{10}{g_m} = \frac{10}{5\text{ mS}} = 2\text{ k}\Omega$$

Substitute $r_{ds} = 50\text{ k}\Omega$

$$(r_{ds}||R_D) = \frac{r_{ds}.R_D}{r_{ds} + R_D} = \frac{50\text{ k}\Omega.R_D}{50\text{ k}\Omega + R_D} = 2\text{ k}\Omega$$

$$\rightarrow R_D = \frac{2\text{ k}\Omega\ .50\text{ k}\Omega}{48\text{ k}\Omega} = 2.08\text{ k}\Omega$$

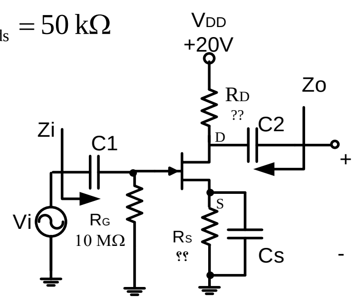
Design Example 2 (Important)

Choose the values of R_D and R_S that will result in voltage gain $|Av| = 8$ using the value of g_m defined at $V_{GSQ} = \frac{1}{4}V_p$

$V_P = -4\text{ V}$

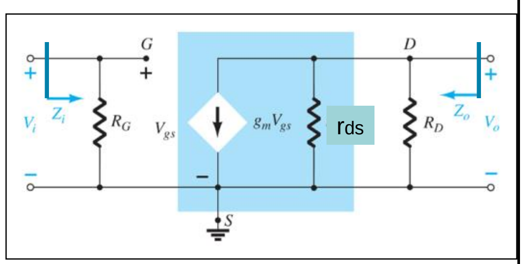
$I_{DSS} = 10\text{ mA}$

$r_{ds} = 50\text{ k}\Omega$



Solution (value of R_D?)

ac ss equivalent circuit



$V_{GS} = \frac{1}{4}V_p = -1$

$I_D = I_{DSS}\left(1 - \frac{-1}{-4}\right)^2 = I_{DSS}.0.5625$

$= 5.625mA$

$g_m = \frac{2I_{DSS}}{|V_P|}\left[1 - \frac{V_{GS}}{V_P}\right]$

$= \frac{2(10\text{mA})}{|-4|}\left[1 - \frac{-1}{-4}\right] = 3.75\text{ m}S$

$V_{gs} = V_i$

$A_v = \frac{V_o}{V_i}$

$V_o = -g_m V_{gs}(r_{ds}||R_D)$

$V_o = -g_m V_i(r_{ds}||R_D)$

$|A_v| = \left|\frac{V_o}{V_i}\right| = |-g_m(r_{ds}||R_D)|$

Since Av & gm are known, then

$|A_v| = \left|\frac{V_o}{V_i}\right| = |-g_m(r_{ds}||R_D)| = 8$

$\therefore (r_{ds}||R_D) = \frac{8}{g_m} = \frac{8}{3.75\text{ mS}} = 2.133\text{ k}\Omega$

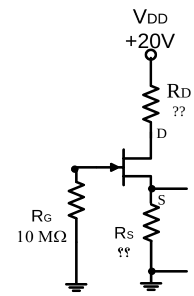
Substitute $r_{ds} = 50\text{ k}\Omega$

$(r_{ds}||R_D) = \frac{r_{ds}.R_D}{r_{ds} + R_D} = \frac{50\text{ k}\Omega.R_D}{50\text{ k}\Omega + R_D} = 2.133\text{ k}\Omega$

$\rightarrow R_D = \frac{2.133\text{ k}\Omega\ .50\text{ k}\Omega}{47.867\text{ k}\Omega} = 2.22\text{ k}\Omega$

Value of Rs?

The value of R_S is determined from DC analysis



Given

$$V_{GS} = V_G - V_S = \frac{1}{4}\mathrm{V}_p = -1$$

$$V_G = 0$$

$$V_S = I_D R_S = 1$$

$$but\ I_D = I_{DSS}\left(1 - \frac{-1}{-4}\right)^2 = I_{DSS}.0.5625 = 5.625mA$$

$$\therefore R_S = \frac{V_S}{I_D} = \frac{1\ \mathrm{V}}{5.625\ \mathrm{mA}} = 177.8\ \Omega$$

Design Example 3

Choose the values of R_D and R_S that will result in

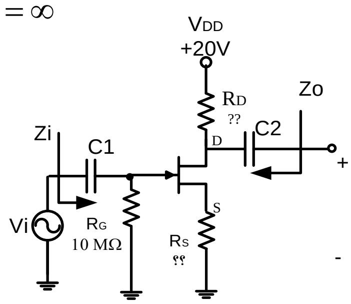
voltage gain $|Av| = 8$ using the value of g_m defined at $V_{GSQ} = \frac{1}{4}V_p$

$V_p = -4\ \mathrm{V}$

$I_{DSS} = 10\ \mathrm{mA}$

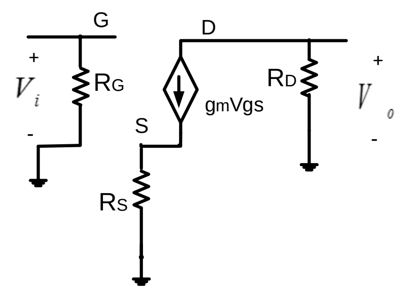
$r_{ds} = \infty$

Note: This is the same previous example except that no Cs (source capacitor)



Solution

ac ss equivalent circuit



$V_{GS} = -1\,\text{V}$

$I_D = 5.625 mA$

$g_m = 3.75\ \text{m}S$ (from previous example)

$A_v = \frac{V_o}{V_i}$

$V_o = -g_m V_{gs}(r_{ds}||R_D)$

$V_{gs} = V_g - g_m V_{gs} R_S$

$V_g = V_i$

$V_{gs} = V_i - g_m V_{gs} R_S$

$V_i = V_{gs} + g_m V_{gs} R_S$

$$V_o = \frac{-g_m V_{gs}(R_D)}{V_{gs} + g_m V_{gs} R_S} = \frac{-g_m R_D}{1 + g_m R_S}$$

$$|A_v| = \left|\frac{V_o}{V_i}\right| = \left|\frac{-g_m R_D}{1 + g_m R_S}\right| = 8$$

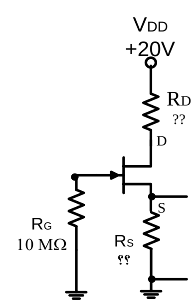
Since Av & gm and R_S are known , then

$R_S = 180\,\Omega$ (based on DC analysis)

$\therefore R_D = 3.573\,\text{k}\Omega$

Value of Rs?

The value of R_S is determined from DC analysis



Given

$$V_{GS} = V_G - V_S = \frac{1}{4}V_p = -1$$

$$V_G = 0$$

$$V_S = I_D R_S = -1$$

$$but\ I_D = I_{DSS}\left(1 - \frac{-1}{-4}\right)^2 = I_{DSS}.0.5625 = 5.625 mA$$

$$\therefore R_S = \frac{V_S}{I_D} = \frac{1\ \text{V}}{5.625\ \text{mA}} = 177.8\ \Omega$$

choose standard value 180Ω